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Natural convection heat transfer in an enclosure with a heated backward step

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Abstract

A numerical analysis was carried out to study the laminar natural convection in an enclosure induced by a heated backward step. The effects of the Rayleigh number Ra, the Prandtl number Pr, and geometrical size of the enclosure on the flow structure and heat transfer characteristics are investigated in detail. Results reveal that the presence of the backward step can enhance the heat transfer coefficient about 32% for $Ra = 10^7$ and $Pr = 0.71$. The extent of heat transfer enhancement increases with the decrease in Ra. The influence of the dimensionless distance between the heated backward step and cooled plate L_2 on the average Nusselt number $\overline{Nu}$ is more significant for the system with a smaller Ra. A larger dimensionless height of the insulated vertical plate H_2 results in a higher $\overline{Nu}$. For $H_2 = L_1 = L_2 = 1$, $Pr = 0.71$, and $10^3 \leqslant Ra \leqslant 10^7$, the average Nusselt number can be correlated as $\overline{Nu} = 0.353Ra^{0.256}$.

Keywords: Natural convection; Enclosure; Backward step; Heat transfer enhancement

1. Introduction

Natural convection heat transfer in enclosures is encountered in many practical situations, such as cooling of electronic equipment by natural convection, heat loss from a solar collector, convection in building elements, convection during the growth of crystals and fluid-filled thermal storage tanks. Over the years, considerable researchers have contributed their efforts to investigate the characteristics of flow structure and heat transfer in the rectangular or square enclosures with two walls at different imposed temperatures. Deviating to these basic geometrical and thermal conditions, enclosures with backward step and the walls subjected to abrupt temperature nonuniformities often occur in applications. The main objective of this study is to examine the natural convection in an enclosure induced by a heated backward step. In addition, the influences of the relative position of the hot and cold wall regions on the transfer behaviors in this enclosure are also discussed.

A lot of articles have dealt with the natural convection problems in the rectangular or square enclosures subjected to a temperature difference in the past. Comprehensive reviews of these typical problems have been done by Ostrach [1], Bejan [2] and Yang [3]. Wroblewski and Joshi [4] numerically studied the laminar heat transfer and flow field arising from a heated protruding element mounted on one vertical wall of an enclosure. Heindel et al. [5,6] theoretically and experimentally investigated the natural convection in a rectangular cavity induced by 3×3 array of discrete, flush-mounted heat sources. They found that the difference between two- and three-dimensional predictions is within 5% as the aspect ratio of heat source is greater than 3. For a liquid-filled square cavity embedded in a substrate, Sathe and Joshi [7,8] studied the characteristics of heat removal from a protruding heat source. Anderson and Bohn [9] presented

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Nomenclature

g	acceleration of gravity
h	heat transfer coefficient
h_1	height of heated backward step
h_2	height of insulated vertical wall
H_2	dimensionless height of insulated vertical wall, h_2/h_1
ℓ_1	distance between insulated vertical wall and heated backward step
L_1	dimensionless distance, ℓ_1/h_1
ℓ_2	distance between heated backward step and cooled plate
L_2	dimensionless distance, ℓ_2/h_1
k	thermal conductivity
Nu	Nusselt number for heated backward step, Eq. (5)
$\overline{Nu}$	average Nusselt number for heated backward step, Eq. (6)
p	pressure
p_∞	static pressure
P	dimensionless pressure, $(p - p_\infty)/(\rho\alpha^2/h_1^2)$
Pr	Prandtl number, ν/α
Ra	Rayleigh number, $g\beta(T_H - T_L)h_1^3/(\nu\alpha)$
T	temperature
T_L	temperature of cooled plate
T_H	temperature of heated backward step
u	horizontal velocity
U	dimensional horizontal velocity, $u/(\alpha/h_1)$
v	vertical velocity
V	dimensionless vertical velocity, $v/(\alpha/h_1)$
x	horizontal coordinate
X	dimensionless horizontal coordinate, x/h_1
y	vertical coordinate
Y	dimensionless vertical coordinate, y/h_1
α	thermal diffusivity
β	coefficient of thermal expansion
θ	dimensionless temperature, $(T - T_L)/(T_H - T_L)$
ρ	density of fluid
ν	kinematic viscosity

the results of an experimental investigation of the influences of surface roughness on natural convection in a cubical enclosure. They obtained a maximum 15% increase in overall Nusselt number. Amin [10] examined the natural convection heat transfer in a two-dimensional vertical enclosure fitted with a periodic array of large rectangular elements on the bottom adiabatic wall. As compared with the results of smooth-wall enclosure, the roughness elements significantly reduce the heat transfer rate across the enclosure. In a later study, Amin [11] stated that if this enclosure with a periodic array of roughness elements is heated from the bottom wall, the heat transfer rate can be greatly enhanced.

In spite of its importance in many practical applications, natural convection in enclosures with a backward step received much less attention than that in open channels. Lin et al. [12,13] and Abu-Mulaweh et al. [14,15] reported measurements and predictions of mixed convection characteristics for open channels with a backward-facing step. In addition, the effects of a backward-facing step on natural convection in open channels were investigated by Abu-Mulaweh et al. [16,17]. Their results reveal that the step height can significantly affect the thermal and hydrodynamic behaviors.

The lack of information on the natural convective heat transfer in enclosures with a backward step motivates the present work. In this study, a numerical analysis is performed to examine the effects of a heated backward step on natural convection in enclosures. Great attention is given to examine the difference in heat transfer characteristics of enclosures with and without the presence of a backward step.

2. Mathematical formulation

The physical system under consideration, as shown schematically in Fig. 1, is a two-dimensional enclosure with a heated backward step located at the left corner. The backward step is kept at a higher, uniform and constant temperature T_H. In the meanwhile, the right wall of the enclosure is at a lower temperature T_L. Besides, the other walls of the enclosure are thermally insulated. Circulating flow in the enclosure is induced by the buoyant force resulting from the given temperature difference between the heated backward step and the cooled plate. In this study, the natural convection flows of Newtonian fluid in enclosures of interest are considered to be steady-state, two-dimensional and laminar. To simplify the analysis, the Boussinesq approximation

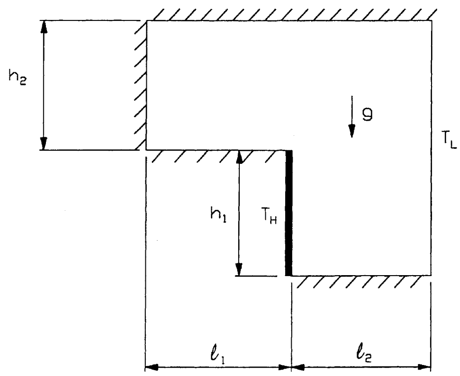

Fig. 1. Schematic diagram of physical system.

is employed to account for the thermal buoyancy effects. According to the stated assumptions, the basic equations in dimensionless form are as follows:

Continuity equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{1}$$

X-momentum equation

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\nabla^2 U, \tag{2}$$

Y-momentum equation

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\nabla^2 V+RaPr\theta, \tag{3}$$

Energy equation

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=Pr\nabla^2 \theta. \tag{4}$$

The corresponding boundary conditions are $\theta = 0$ and 1 for the cold and hot plates, respectively. The normal temperature gradients are zero for all the other surfaces. In addition, no-slip condition is applied to all solid walls.

The local and average Nusselt numbers for the heated backward step are of interest to the thermal system design. These are defined as

$$Nu=\frac{hh_1}{k}=\frac{h_1(-\partial T/\partial x)_{x=\ell_1}}{T_\mathrm{H}-T_\mathrm{L}}=-\left(\frac{\partial\theta}{\partial X}\right)_{X=L_1} \tag{5}$$

and

$$\overline{Nu}=\int_0^1 Nu\,\mathrm{d}Y. \tag{6}$$

3. Numerical method

The power-law scheme is used to transform the above governing equations, Eqs. (1)–(4), into finite difference equations which, then, are solved by the ADI method. The variables U, V, P and θ are distributed on staggered grids and the SIMPLER scheme [18] is adopted to solve the coupled equations. Non-uniform grids are employed and more gridlines are concentrated beside the walls. The solutions are considered to be convergent if the relative errors of consecutive iteration are less than 10^{-4} for U, V, P and θ, and the relative difference of input and output energy is within 0.5%.

The numerical code employed for the present study was rigorously verified. First, a computation experiment was carried out to ensure that the solution is grid-independent. For a typical case with $Ra = 10^7$, $Pr = 0.71$, and $L_1 = L_2 = H_2 = 1$, it is found that the differences in velocity, temperature and average Nusselt number for 80×80 and 110×110 grids are always less than 1.2%. Accordingly, the 80×80 grid is considered to be suitable for the present study. To further check the adequacy of the numerical scheme, results for a limiting case of natural convection in a square enclosure were obtained. Excellent agreement was found between the present predicted results and those of de Vahl Davis [19] and Quere [20], as shown in Table 1. Another limiting case is the natural convection in a rectangular enclosure in which the heated surface is located at the lower-half portion of one vertical wall and the cooled surface is at the upper-half portion of the other vertical wall. Table 2 lists the results of present prediction and those of Valencia and Frederick [21]. It can be observed from Table 2 that the difference is within 3% for $Ra \leqslant 1.25 \times 10^4$, and about 17% for $Ra = 1.25 \times 10^6$. It should be pointed out that an uniform grid system with 20×20 (i.e., $\Delta X = \Delta Y = 0.05$) was used by Valencia and Frederick [21]. Their average Nusselt number for $Ra = 1.25 \times 10^5$ is 12.2% higher than that of [22] when the values are extrapolated to zero finite element size. Besides, the results of de Vahl Davis [19] show that the average Nusselt number for $\Delta X = \Delta Y = 0.05$ is 4.5% higher than that for $\Delta X = \Delta Y = 0.0125$ as $Ra = 10^6$. Thus the large deviation in $\overline{Nu}$ for case with $Ra = 1.25 \times 10^6$ between the present study and that of Valencia and Frederick [21] may be attributed to the fact that the results in [21] were over-predicted owing to the comparatively rougher gridlines near the walls. These verifications lead to strong support of the use of the present numerical scheme.

Table 1
Comparison of predicted Nusselt numbers with those of de Vahl Davis [19] and Quere [20] under various Rayleigh numbers

Ra	$\overline{Nu}$		
	de Vahl Davis [19]	Quere [20]	Present study
10^3	1.117		1.118
10^4	2.238		2.243
10^5	4.509		4.514
10^6	8.817	8.825	8.805
10^7		16.52	16.46
10^8		30.23	30.11

Table 2
Comparison of predicted Nusselt numbers with those of Valencia and Frederick [21] under various Rayleigh numbers

Ra	$\overline{Nu}$	
	Valencia and Frederick [21]	Present study
1.25×10^2	0.73	0.74
1.25×10^3	1.50	1.48
1.25×10^4	2.86	2.76
1.25×10^5	5.83	5.21
1.25×10^6	11.72	9.70

4. Results and discussion

Inspection of the foregoing analysis discloses that the characteristics of natural convection in an enclosure induced by a heated backward step depends on five governing parameters. These are the Rayleigh number *Ra*, the Prandtl number *Pr*, the dimensionless height of the insulated vertical wall H_2, the dimensionless distance between the insulated vertical wall and heated backward step L_1, and the dimensionless distance between heated backward step and cooled wall L_2. In the following, we present the results for air ($Pr = 0.71$) and water ($Pr = 7.0$) in the enclosure with *Ra* varying from 10^2 to 10^7, H_2 from 0.3 to 2, L_1 from 0.1 to 1.5, and L_2 from 0.3 to 5. The effects of these parameters on the flow structure and thermal behavior are investigated in detail. Additionally, the differences in the results for the enclosures with or without backward step effects will be presented.

The isotherms and streamlines for air flow ($Pr = 0.71$) with $Ra = 10^2$ and $H_2 = L_1 = L_2 = 1$ are illustrated in Fig. 2. As can be seen in Fig. 2(a), the isotherms horizontally distribute from the left plate to the cold plate and are parallel to each other in the bottom region. This indicates that the conduction plays the dominant role for heat transfer process. However, the isotherms are gradually distorted toward left with increasing *Y*. The phenomena are particularly evident in the region above backward step. Accordingly, the existence of a backward step can significantly affect the heat transfer characteristics. The streamlines plotted in Fig. 2(b) reveal that only a large recirculation cell is formed and slightly expands into the upper space of the backward step. This is due to the fact that the induced flow is rather weak under such a low Rayleigh number ($Ra = 10^2$). On further increasing *Ra*, it is expected that the buoyancy-induced flow becomes stronger and the convective heat transfer becomes more important. Shown in Fig. 3 are the isotherms and streamlines for $Ra = 10^7$. It is noted in Fig. 3(a) that two extremely thin boundary layers are found near the two heat transfer plates. Thermal stratification takes place in the most area between the heated and cooled plates. In addition, a rather large portion of the space above the backward step remains nearly isothermal. These features are rather different from those for $Ra = 10^2$. The streamlines in Fig. 3(b) show that a secondary cell forms at $Y > 1$ and two tiny cells are found at the left corners. Comparison of Figs. 2(b) and 3(b) discloses that the value of the streamfunction for $Ra = 10^7$ is several hundred times larger than that for $Ra = 10^2$.

The variations of the average Nusselt number $\overline{Nu}$ with different *Ra* and *Pr* are plotted in Fig. 4. To illustrate the effects of a backward step on the heat transfer characteristics, the results of de Vahl Davis [19] and Quere [20] for the enclosure without backward step are also presented. As shown in this figure, better heat transfer is noted for a system with a backward step. For example, the Nusselt number $\overline{Nu}$ has 32% difference for the cases with or without a backward step as $Ra = 10^7$ and $H_2 = L_1 = L_2 = 1$. Additionally, the magnitude of enhancement increases with a decrease in *Ra*. It is also noted in Fig. 4 that the effects of working fluid, water ($Pr = 7$) and air ($Pr = 0.71$), on the average Nusselt number are insignificant. To our knowledge, the independence of heat transfer on the Prandtl number has been numerically and experimentally reported by several researchers for different configurations. For example, Incropera et al. [23] conducted an experiment to investigate the force convection heat transfer from discrete heat sources in a channel. They observed negligible difference between the results for $Pr = 5.4$ and $Pr = 22.1$.

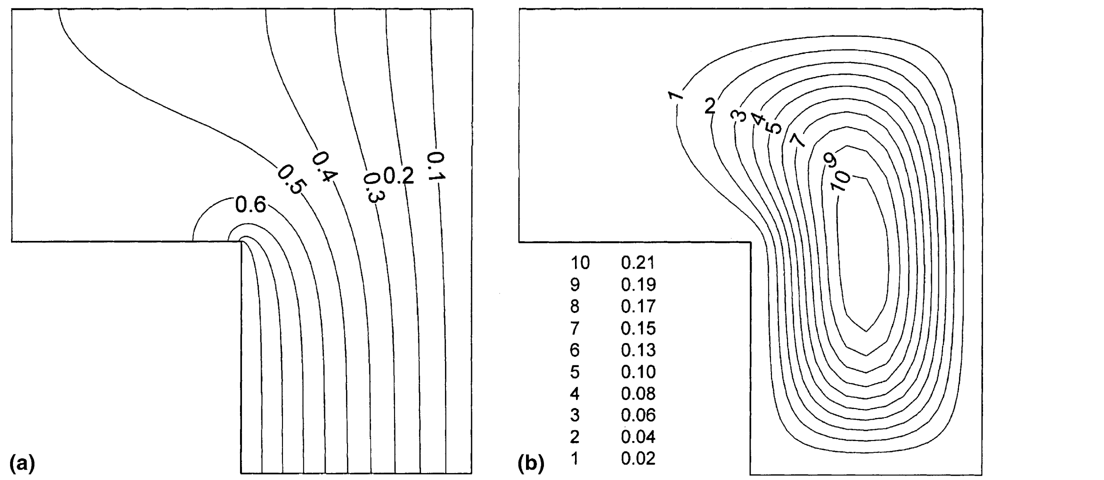

Fig. 2. Plots of (a) isothermal and (b) stream function contours for $Ra = 10^2$, $Pr = 0.71$ and $H_2 = L_1 = L_2 = 1$.

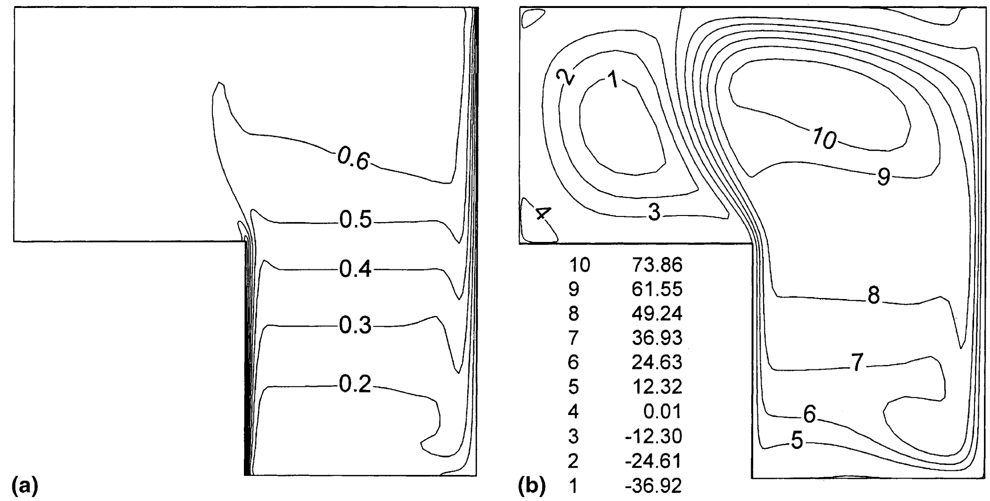

Fig. 3. Plots of (a) isothermal and (b) stream function contours for $Ra = 10^7$, $Pr = 0.71$ and $H_2 = L_1 = L_2 = 1$.

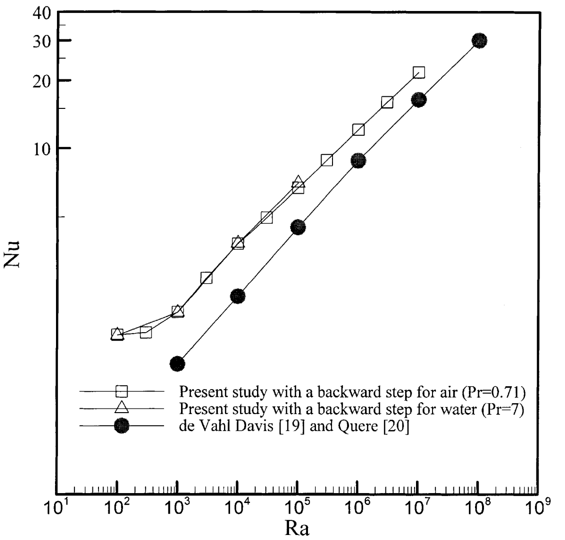

Fig. 4. Variations of average Nusselt number versus Rayleigh number for air ($Pr = 0.71$) and water ($Pr = 7$).

Heindel et al. [6] and Keyhani et al. [24,25] numerically and experimentally examined the natural convection in vertical cavity with discrete heat sources. The fluids with Prandtl number varying from 1 to 150 were examined. They claimed that the effects of Prandtl number on the average Nusselt number are rather insignificant.

Next, the dependence of heat transfer and flow characteristics on changing the distance between the heated backward step and cooled plates L_2 is examined. For the case with $Ra = 10^4$ and $L_2 = 0.3$, the isotherms plotted in Fig. 5 are nearly parallel to each other in the narrow gap. This indicates that conduction is the dominant heat transfer mechanism. Moreover, drastic distortion of the isotherms is found at $Y > 1$, which is resulted from the large recirculation above the backward step. As the Rayleigh number (based on the height of heated backward step) is fixed at $Ra = 10^4$, the numerical results illustrate that the heat convection can be gradually getting important with the increase in L_2. Shown in Fig. 6 are the isotherms and streamlines for the case with $Ra = 10^4$ and $L_2 = 5$. It is seen in Fig. 6 that thermal stratification becomes apparent. Fig. 7 summarizes the effect of L_2 on the variations of average Nusselt numbers. When $Ra = 10^4$, $\overline{Nu}$ equals 4.02 for $L_2 = 0.3$ and decreases monotonically with the increase in L_2, for example, $\overline{Nu} = 2.92$ for $L_2 = 5$. This fact can be made plausible by noting the fact that the conductive heat transfer is the dominant heat transfer mode for a system with a small Rayleigh number ($Ra = 10^4$) and a short distance between the heated and cooled plates. However, the conductive heat transfer is inversely proportional to the distance between the hot and cold plates (L_2). Although the convective heat transfer can be augmented as L_2 is enlarged, the enhancement of convective heat transfer resulting from the increasing in L_2 is too weak to compensate the reduction of conductive heat transfer. On the other hand, for $Ra = 10^7$ the convective heat transfer plays such an important role that the conductive heat transfer is negligible. Additionally, the strong buoyancy results in very thin thermal boundary layers. Therefore, the effects of L_2 on the thickness of thermal boundary layer is rather insubstantial. Consequently, the effect of L_2 on the average Nusselt number $\overline{Nu}$ is insignificant for $Ra = 10^7$.

Attention is now turned to examine the effects of the dimensionless height above the backward step H_2 on the

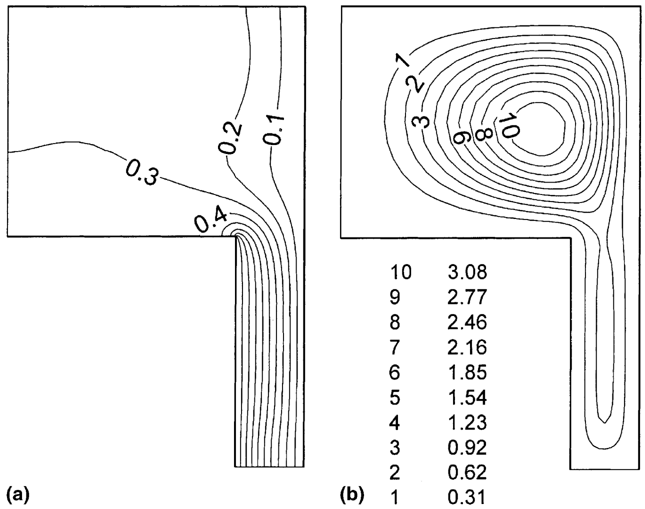

Fig. 5. Plots of (a) isothermal and (b) stream function contours for $Ra = 10^4$, $Pr = 0.71$, $H_2 = L_1 = 1$ and $L_2 = 0.3$.

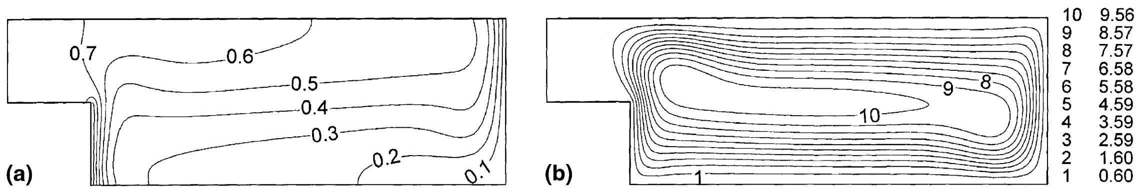

Fig. 6. Plots of (a) isothermal and (b) stream function contours for $Ra = 10^4$, $Pr = 0.71$, $H_2 = L_1 = 1$ and $L_2 = 5$.

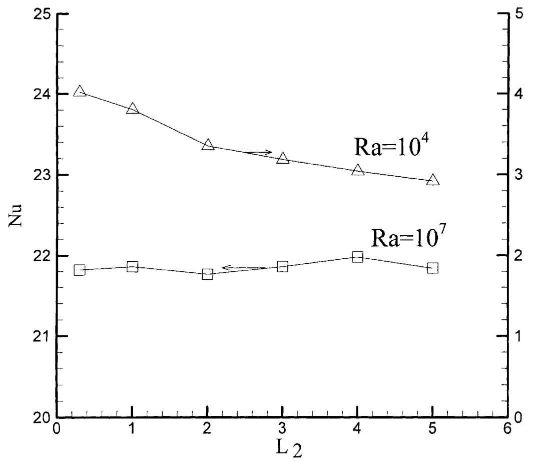

Fig. 7. The effect of L_2 on the variations of average Nusselt numbers for $H_2 = L_1 = 1$.

heat transfer behaviors. The comparison of the isotherms for $H_2 = 2$ and 0.3 is shown in Figs. 8(a) and (b). For the system with a higher H_2, the thickness of thermal boundary layer is thinner and the distortion of isotherms in the region near the backward step is more drastic. This phenomenon is due to the fact that the induced natural convection is stronger and more fluid can enter the left gap for a higher H_2. In addition, the prediction results provide that $\overline{Nu}$ is 23.11 and 18.9 for $H_2 = 2$ and 0.3, respectively.

The previous results have illustrated the effects of heated backward step on natural convection heat transfer in an enclosure with the whole right vertical wall being subjected to a cooled condition. It is interesting to investigate the characteristics of flow structure and heat transfer for the enclosure with the right vertical wall being partially insulated, while the right vertical wall is divided into two equal portions and two kinds of thermal boundary arrangements are considered. One is that the upper-half portion of the right vertical wall is thermally insulated and the lower-half portion is kept at a low temperature. This means $\theta = 0$ for $0 < Y < 1$ and $\partial\theta/\partial X = 0$ for $1 < Y < 2$. The other arrangement is that the lower portion of the wall is thermally insulated and the upper portion is maintained at a low temperature. This is $\partial\theta/\partial X = 0$ for $0 < Y < 1$ and $\theta = 0$ for $1 < Y < 2$. Fig. 9 presents the isotherm and stream function plots for the enclosure with the lower portion of the right vertical wall being subjected to a cooled condition. The isotherms in Fig. 9(a) reveal that a large

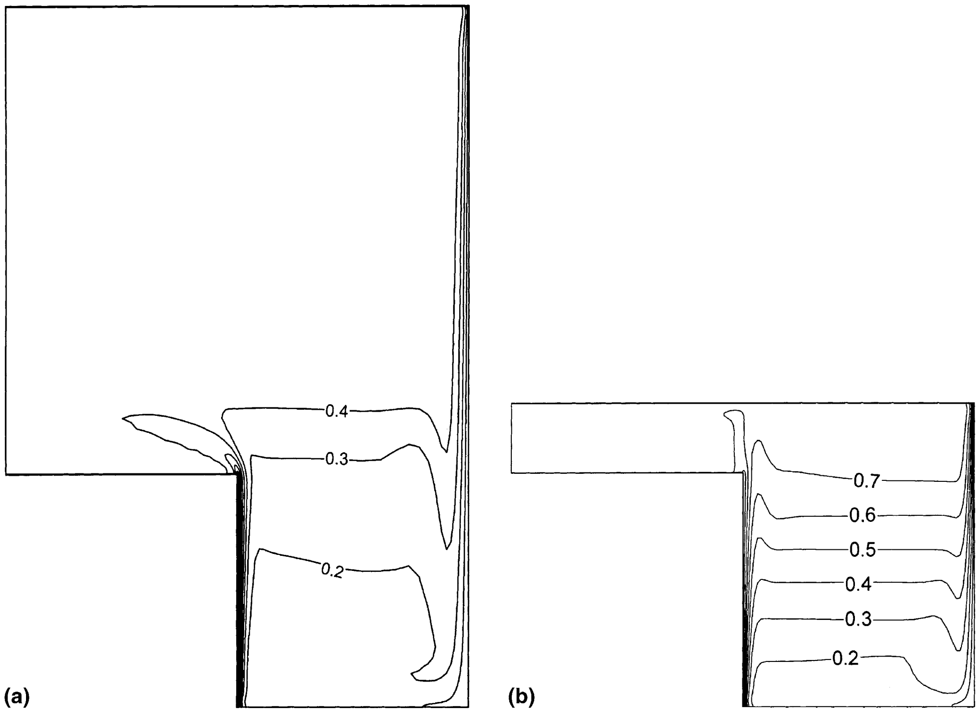

Fig. 8. Plots of isothermal contours for $Ra = 10^7$, $Pr = 0.71$, $L_1 = L_2 = 1$ and (a) $H_2 = 2$ and (b) $H_2 = 0.3$.

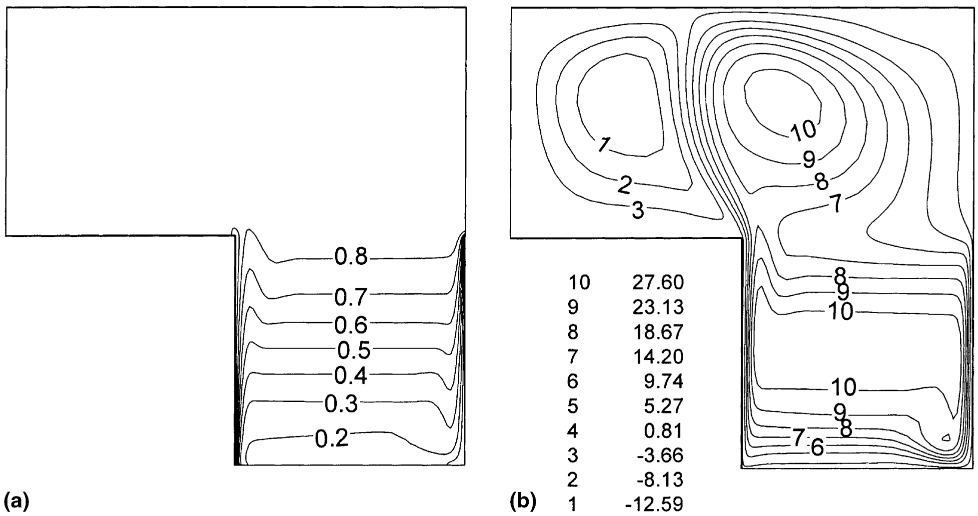

Fig. 9. Plots of (a) isothermal and (b) stream function contours for $Ra = 10^6$, $Pr = 0.71$, $H_2 = L_1 = L_2 = 1$. $\theta = 0$ for $0 < Y < 1$ and $\partial\theta/\partial X = 0$ for $1 < Y < 2$ at $X = 2$.

isothermal region exists in $Y > 1$. Fluid is then subjected to negligible thermal buoyancy force and, consequently, circular streamlines are found in this region. However, the flow and thermal features in the region of $Y < 1$ are very similar to those of benchmark solution [19]. Fig. 10 depicts the results for the situation with the upper-half portion of the right vertical wall being the cooling surface. It is seen that there are two nearly isothermal regions located at $Y > 1$ and $Y < 0.5$. Between them, there exists a temperature-stratified region, but the vertical temperature gradient is much smaller than that at $Y < 1$ in Fig. 9(a). In addition, the stream function range listed

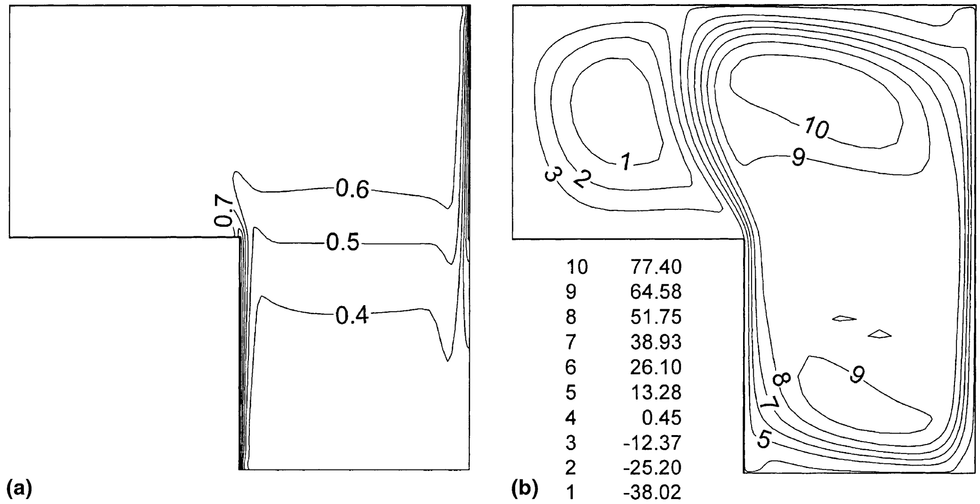

Fig. 10. Plots of (a) isothermal and (b) stream function contours for $Ra = 10^6$, $Pr = 0.71$, $H_2 = L_1 = L_2 = 1$. $\partial\theta/\partial X = 0$ for $0 < Y < 1$ and $\theta = 0$ for $1 < Y < 2$ at $X = 2$.

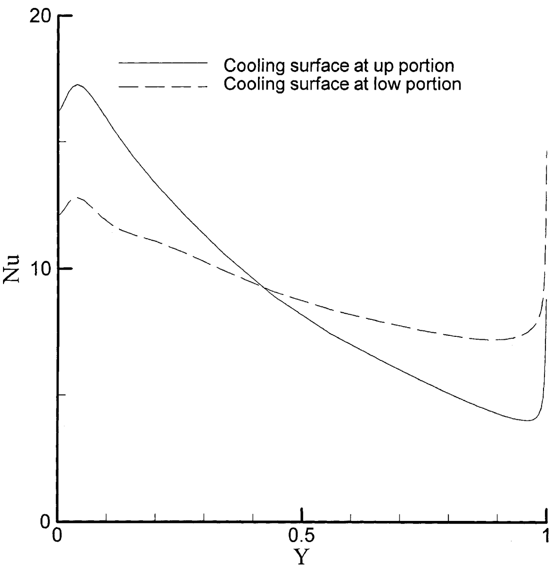

Fig. 11. The effect of two different thermal boundary conditions for the right vertical wall on the distribution of the local Nusselt number.

in Fig. 10(b) is much wider than that in Fig. 9(b). This fact indicates that a strong velocity field is obtained for the situation with the upper-half portion of right vertical wall being the cooling surface. The distributions of the local Nusselt number for the two configurations are plotted in Fig. 11. It is seen that the heat transfer is more evenly distributed when the cooling surface located at the upside. However, we obtain almost identical average Nusselt number for both configurations. This demonstrates that the overall heat transfer is not affected by the different location of the half-cooling wall.

Finally, it should be pointed out that, according to our computational results, the effect of the distance between the left wall and heated plate L_1 on the heat transfer is indistinguishable for $L_1 > 0.1$. When L_1 varies from 0.1 to 1.5, the difference in $\overline{Nu}$ is within 3%.

5. Conclusion

Natural convection in an enclosure with a heated backward step was numerically investigated in the present study. The effects of the Rayleigh number Ra, the Prandtl number Pr, the geometrical size of enclosure and the allocation of cooling surface are examined in detail. What follows is a brief summary of the major results.

1. The presence of a backward step can enhance the average Nusselt number $\overline{Nu}$ by about 32% for $Ra = 10^7$. The extent of heat transfer enhancement increases with the decreasing in Ra.
2. The influence of L_2 on $\overline{Nu}$ is more significant for the system with a smaller Ra. A larger H_2 results in a higher $\overline{Nu}$. Besides, the variations in Pr cause a little effect on $\overline{Nu}$.
3. A correlation of the average Nusselt number is proposed as $\overline{Nu} = 0.353Ra^{0.256}$ for $H_2 = L_1 = L_2 = 1$, $Pr = 0.71$, and $10^3 \leqslant Ra \leqslant 10^7$.
4. As the wall facing the heated backward step is divided into two equal regions, in which one is the cooling surface and the other is thermally insulated, the allocation of the two regions do not significantly affect the average Nusselt number.

Acknowledgements

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